Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Numerical Value	DAILY TUTORIAL SHEET 11

201.(32) $15! = 2^{11}3^{6}5^{3}7^{2}11^{1}13^{1}$

Number of ways = Number of ways 15! Can be expressed as product of 2 relatively prime divisors $= 2^{6-1} = 32$

202.(495) The number of integers will be same as number of non-negative integral solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 8$ i.e. ${}^{12}C_4 = 495$.

203.(12) Let n = number of women $n \times 3 = 9 \times 4 \;\Rightarrow\; n = 12$

204.(48) Guests can be accommodated as shown $\begin{bmatrix} G & X \\ X & G \\ G & X \\ X & G \end{bmatrix} or \begin{bmatrix} X & G \\ G & X \\ X & G \\ G & X \end{bmatrix}$ $\therefore$ number of ways $= 2 \times 4! = 48$

205.(60) A – – A = 4 × 3

A – – A = 4 × 3 × 2

A – – – – A = 4 × 3 × 2 × 1

Total number of ways = 12 + 24 + 24 = 60

206.(7) Number of ways $= (n-2)!\,{}^{n-1}C_2 2! = (n-2)(n-1)!$

$3600 = 5 \times 6! \qquad \Rightarrow \qquad n = 7$

207.(276) $x < z < y$

$x < z = y$

$x = z < y$

$z < x < y$

Number of ways $= {}^{9}C_3 + {}^{9}C_2 + {}^{9}C_2 + {}^{10}C_3 = 2\left({}^{10}C_3\right) + {}^{9}C_2 = 276$

208.(56) Different sequence like 122, 222, 345, etc are possible. No. of sets = no. of ways to distribute 3 alike dice over 6 numbers 1, 2, 3, 4, 5, 6 with repetitions allowed $= {}^{3+6-1}C_{6-1} = {}^{8}C_5 = 56$

209.(972) $*(a) = 7 \qquad \Rightarrow \qquad a = P_1^6$

$*(g) = 13 \qquad \Rightarrow \qquad g = P_2^{12}, \quad P_1, P_2$ are prime

$g = ar^6 \qquad \Rightarrow \qquad p_1^6 r^6 = P_2^{12} \;\Rightarrow\; r = \dfrac{P_2^2}{P_1}$

$$632 = d - c = P_1^6\left(r^3 - r^2\right) = P_1^6\left(\frac{P_2^6}{P_1^3} - \frac{P_2^4}{P_1^2}\right) = P_1^3P_2^6 - P_1^4P_2^4$$

$\Rightarrow \quad 2^4 3^3 = P_1^3 P_2^4\left(P_2^2 - P_1\right) \qquad \Rightarrow \qquad P_2 = 2, P_1 = 3 \qquad \Rightarrow \qquad b = 972$

210.(7) There are 26 number each of the form 4n. $4n+2, 4n+3.$ and 27 numbers of the form $4n+1$ in the given set $N = \underbrace{\left(3\,{}^{26}C_4 + {}^{27}C_4\right)}_{0000,1111,2222,3333} + \underbrace{\left({}^{26}C_2\,{}^{27}C_1\,{}^{26}C_1\right)}_{0013} + \underbrace{\left({}^{26}C_2\,{}^{26}C_2\right)}_{0022} + \underbrace{\left({}^{27}C_2\,{}^{26}C_2\right)}_{1133} + \underbrace{\left({}^{26}C_2\,{}^{27}C_1\,{}^{26}C_1\right)}_{2213}$

= 738400

Number below the bracket remainders left by four chosen numbers.

211.(1) 1_ _ _ _ _ $= 5 \times 5! = 600$

2_ _ _ _ _ $= 5 \times 5! = 600$

3_ _ _ _ _ $= 5 \times 5! = 600$

41 _ _ _ _ $= 4 \times 4! = 96$

42_ _ _ _ _ $= 4 \times 4! = 96$

$431\_\,\_\,\_\,\_ = 3 \times 3! = 18$

$4321\_\,\_\,\_ = 2!$

$43256\_\,\_ = 2!$

2018^{th} number is 4325671

212.(5) 6 A like $\rightarrow$ 5

4 A like + 2 others alike $\rightarrow 5 \times 4\ \dfrac{6!}{4!2!} = 300$

3 Alike + 3 others alike $\rightarrow\ {}^5C_2 \times \dfrac{6!}{3!3!} = 200$

2 Alike + 2 other alike + 2 other alike $\rightarrow\ {}^5C_3 \dfrac{6!}{2!2!2!} = 900$

N = 900 + 200 + 300 + 5 = 1405

213.(120) Let $x_1x_2x_3x_4x_5x_6x_7x_8$ be the eight-digit number

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 4$

$x_1 \geq 1$ & $x_i \geq 0\ i > 1\ x_i \in I \qquad \Rightarrow \qquad N = {}^{4-1+7}C_7 = {}^{10}C_7 = 120$

214.(70) $1000 = 2^3 5^3$, $2000 = 2^4 5^3$

a	b	c
2^3	$2^0, 2^1, 2^2, 2^3$	2^4
$2^0, 2^1, 2^2$	2^3	2^4
		= 7 ways
5^3	$5^0, 5^1, 5^2$	5^3
5^3	5^3	$5^0, 5^1, 5^2, 5^3$
$5^0, 5^1, 5^2$	5^3	5^3
		= 10 ways

Number of ways = 7 × 10 = 70

215.(45) Number of dolls each girl has got = number of times each girl goes $= {}^6C_4 = 15$

$\therefore$ Total number of dolls = 15 × 3 = 45